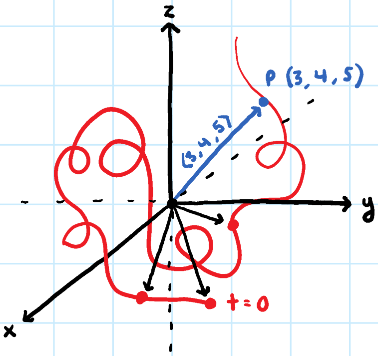
Trajectories

Wednesday, May 17, 2023 8:56 AM

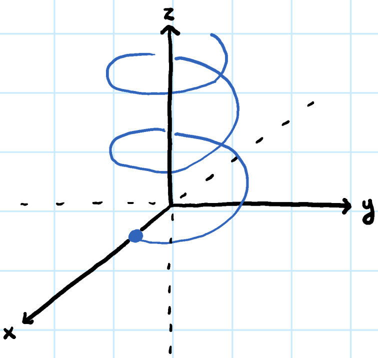
trajectory : $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

functions of variable t



* describes curve, as well as how fast going thru curve *

ex 1) $\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle$



helix (spiral)

where is particle @...

$t=0$ $\quad \vec{r}(0) = \langle 1, 0, 0 \rangle$

$t=1$ $\quad \vec{r}(1) = \langle \cos(1), \sin(1), 1 \rangle$

$t=2\pi$ $\quad \vec{r}(2\pi) = \langle 1, 0, 2\pi \rangle$

thm: given line L thru point $P = (p_1, p_2, p_3)$ & vector direction $\vec{v} = \langle v_1, v_2, v_3 \rangle$

the trajectory $\vec{r}(t)$ of particle along line L is:

$$* \quad \vec{r}(t) = \langle p_1 + tv_1, p_2 + tv_2, p_3 + tv_3 \rangle \quad *$$
$$= \vec{OP} + t \cdot \vec{v}$$

* not linear equation → most likely not line *

ex 2) find parametric description of $\vec{r}(t)$, a trajectory along line thru $\overbrace{(1,2,-3)}^{P}$ & $\vec{v} = \langle -1, 0, 1 \rangle$

form $f(t)$

solution: by thm...

$$\vec{r}(t) = \vec{OP} + t\vec{v} = \langle \underbrace{1-t}_{x(t)}, \underbrace{2}_{y(t)}, \underbrace{-3+t}_{z(t)} \rangle$$

def: given trajectory $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

the velocity of $\vec{v}(t)$ is derivative :

could be written as dot

$$\vec{v}(t) = \vec{r}(t)' = \langle x(t)', y(t)', z(t)' \rangle = \frac{d\vec{r}(t)}{dt} \quad \text{(vector)}$$

the speed is :

$$|\vec{v}(t)| = \sqrt{x(t)'^2 + y(t)'^2 + z(t)'^2} \quad (\#)$$

meters (m)

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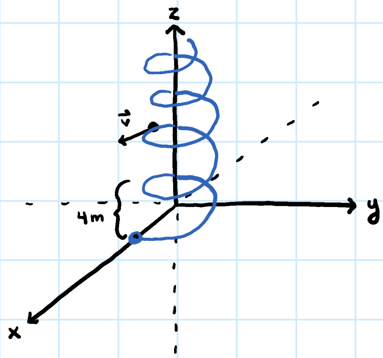
ex 3) $\vec{r}(t) = \langle \cos(t), \sin(t), 4t^2 \rangle$, find velocity & speed @ $t = 2$

solution:

$$\vec{v}(t) = \vec{r}(t)' = \langle -\sin(t), \cos(t), 8t \rangle$$

thus @ $t = 2$: $\boxed{\vec{v}(2) = \langle -\sin(2), \cos(2), 16 \rangle}$



* traveling faster *

speed @ $t = 2$:

$$|\vec{v}(2)| = \sqrt{\underbrace{-\sin(2)^2 + \cos(2)^2}_{\sin(x)^2 + \cos(x)^2 = 1} + 16^2} = \sqrt{1 + 16^2} = \boxed{\sqrt{257} \ m/s}$$